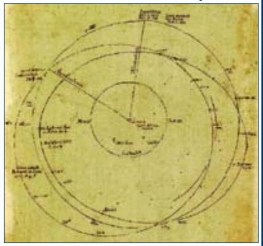
Relative Angles Only Orbit Determination Using Gauss's Method

Introduction

On January 1, 1801, the Italian astronomer Joseph Piazzi discovered a planetoid, working from an observatory in Palermo, Italy. This object, which he christened Ceres, was moving in the constellation Taurus. Astronomers were only able to observe the planetoid for forty-one days, during which it's orbit swept out an angle of only nine degrees. The observations consisted of two angle measurements (right ascension and declination) and the time at which the observation took place. Ceres was then lost to sight when its light vanished in the rays of the sun, and the astronomers could no longer find it. Piazzi's observations of Ceres were published in June of 1801, and most leading astronomers in Europe were scrambling to determine its orbit. Carl Friedrich Gauss was born in 1777 in Brunswick Germany. At this point, Gauss had already worked with astronomical questions, such as the theory of the motion of the moon, and had developed a method which would allow for the determination of Ceres' orbit. Gauss performed his calculations, which required over one hundred hours of work, and the results were published in September of 1801. When Ceres reappeared according to Gauss's predictions the mathematician and physicist gained great renown. Gauss had shown that it is possible to determine the orbit of an unknown orbiting object by using angle measurements alone.

Sketch of the Orbit of Ceres by Gauss



The application of Gauss's method to the area of satellite detection and rendezvous has yet to be fully explored, and may yet prove to be an efficient and cost effective method of orbit determination.

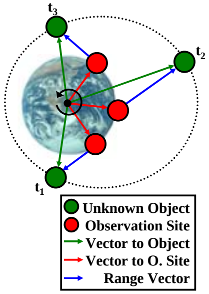
Classical Application of Gauss's Method

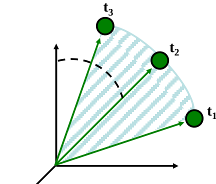
Gauss's Method For Planetary Orbit Determination

- Three separate observations of an unknown object are obtained. These observations consist of azimuth and elevation angles along with the times the observations occurred.
- Azimuth and elevation angles are used to calculate a unit vector from the observation site to the unknown object at each time.
- The position vector to the observation site at each time is obtained. The position vector will change due to the rotation of the Earth.
- The direction of the position vector from the observation site to the unknown object is known from the angle measurements, but the magnitude (or range) is unknown and must be solved for.
- It is assumed that the orbit of the unknown object lies within a single plane. This assumption allows for a linear relationship between the position vectors of the unknown object at the three different times.
- The three position vectors to the unknown object can also be related using Lagrange coefficients.
- Approximate Lagrange coefficients are obtained from a Taylor series expansion of the equations of motion.
- The system is now well posed, having 18 equations and 18 unknowns.

▪A solution can now be obtained for the position vector of the unknown object at each of the three times from which the orbit of the object can be completely described.

The position of the object at each time (t) can be described by vector addition





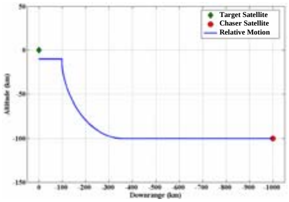
Orbit of the object lies within a single plane

Orbital Rendezvous Scenarios

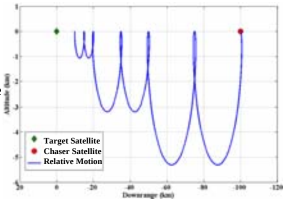
Gauss's Method Can Be Used to Determine the Orbit of an Unknown Satellite.

In the area of orbital rendezvous the angle measurements will be made by another satellite, rather than from a stationary observation site on Earth. The following are orbital rendezvous scenarios in which Gauss's method could be useful in determining the orbit of an unknown satellite.

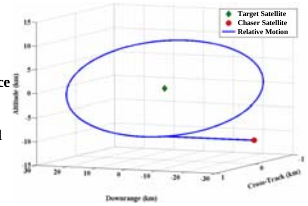
The chaser approaches the target from a lower co-elliptic orbit. The chaser performs a maneuver to place itself at a higher orbit. Once the chaser reaches the desired altitude another maneuver is performed to reinitiate a co-elliptic orbit.



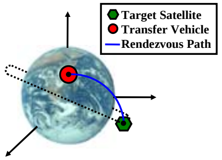
The chaser and the target are initially in the same orbit but are slightly out of phase. The chaser initiates a phasing orbit and appears to "hop" towards the target. The size of the hops decreases as the chaser nears the target. After each hop, the chaser may also initiate a maneuver to remain stationary behind the target for observation purposes.



The chaser approaches the target from a lower co-elliptic orbit. When the chaser is directly below the target a maneuver is performed to place the chaser in a safety ellipse "football" around the target. This football orbit can be used to observe and inspect the target vehicle



A transfer vehicle is launched from Earth to rendezvous with the target satellite in low Earth orbit. The orbit of the target satellite can be determined using Gauss's method. Gauss's method could also be used by the target satellite to determine the orbit of the incoming transfer vehicle in order to avoid rendezvous if so desired.



Rendezvous Application of Gauss's Method

Gauss's Method Can Be Extended to Orbital Rendezvous

- Angle measurements to a satellite of interest can be made by another satellite and not by an observation site on the surface of the earth.
- A large number of satellites are currently equipped with cameras as part of their navigation systems. These cameras can be used to determine the orbit of an unknown satellite.
- If prior knowledge of the orbit is available though some means, Gauss's method can be modified to produce more accurate results.
- Once a preliminary estimate of the orbit is made by Gauss's method, the estimate can be iteratively refined. This will improve the accuracy of the estimate and extend the usefulness of Gauss's Method.

▪After the initial estimate of the orbit is completed a Kalman filter can be used to process subsequent measurements that are made by the satellite. A Kalman filter requires an initial estimate of the location of the object in order to begin the filtering process. This estimate can be provided by Gauss's Method

The primary objective in using Gauss's method in satellite applications is to obtain an initial estimate of the orbit of the satellite of interest. Once this initial orbit has been determined a Kalman filter can be used to process subsequent measurements. A Kalman Filter cannot be started without an initial estimate of the orbit.

Preliminary Orbit Determination Using Gauss's Method

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Iterative Improvement of Preliminary Orbit

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Kalman Filter Processing of Subsequent Measurements

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